

CLASS : XIIth

DATE :

SOLUTIONS

SUBJECT : MATHS

DPP NO. :1

Topic :-DIFFERENTIAL EQUATIONS

1 (a)

The given differential equation can be rewritten as

$$y + \frac{d^2y}{dx^2} = \left[a + \left(\frac{dy}{dx} \right)^{3/2} \right]^2$$

$$\Rightarrow y + \frac{d^2y}{dx^2} = x^2 + \left(\frac{dy}{dx} \right)^3 + 2x \left(\frac{dy}{dx} \right)^{3/2}$$

$$\Rightarrow \left[y + \frac{d^2y}{dx^2} - x^2 - \left(\frac{dy}{dx} \right)^3 \right]^2 = \left[2x \left(\frac{dy}{dx} \right)^{3/2} \right]^2$$

 $\therefore$ Order and degree of the given differential equation is 2 and 2 respectively.

2 (c)

Given differential equation is $\frac{dy}{dx} + y = e^x$

$$\therefore \text{IF} = e^{\int P dx} = e^{\int 1 dx} = e^x$$

Now, solution is

$$ye^x = \int e^{2x} dx$$

$$\Rightarrow ye^x = \frac{e^{2x}}{2} + \frac{c}{2}$$

$$\Rightarrow 2ye^x = e^{2x} + c$$

3 (b)

We have,

$$\phi(x) = \phi'(x)$$

$$\Rightarrow \frac{\phi'(x)}{\phi(x)} = 1$$

$$\Rightarrow \log \phi(x) = x + \log C \Rightarrow \phi(x) = C e^x$$

$$\text{Putting } x = 1, \phi(1) = 2, \text{ we get } C = \frac{2}{e}$$

$$\therefore \phi(x) = 2e^{x-1} \Rightarrow \phi(3) = 2e^2$$

4 (b)

Given equation

$$\frac{dy}{dx} + \sin\left(\frac{x+y}{2}\right) = \sin\left(\frac{x-y}{2}\right)$$

$$\Rightarrow \frac{dy}{dx} = \sin\left(\frac{x-y}{2}\right) - \sin\left(\frac{x+y}{2}\right)$$

$$\Rightarrow \frac{dy}{dx} = -2 \sin\left(\frac{y}{2}\right) \cos\left(\frac{x}{2}\right)$$

$$\Rightarrow \operatorname{cosec}\left(\frac{y}{2}\right) dy = -2 \cos\left(\frac{x}{2}\right) dx$$

On integrating both sides, we get

$$\int \operatorname{cosec}\left(\frac{y}{2}\right) dy = - \int 2 \cos\left(\frac{x}{2}\right) dx + c$$

$$\Rightarrow \frac{\log(\tan \frac{y}{4})}{\frac{1}{2}} = - \frac{2 \sin\left(\frac{x}{2}\right)}{\frac{1}{2}} + c$$

$$\Rightarrow \log(\tan \frac{y}{4}) = c - 2 \sin\left(\frac{x}{2}\right)$$

5 **(a)**

The family of curves is

$$x^2 + y^2 - 2ax = 0 \quad \dots(i)$$

Differentiating w.r.t. to x , we get

$$2x + 2y \frac{dy}{dx} - 2a = 0 \Rightarrow a = x + y \frac{dy}{dx}$$

Substituting the value of a in (i), we obtain

$$x^2 + y^2 - 2x\left(x + y \frac{dy}{dx}\right) = 0 \text{ or, } y^2 - x^2 - 2xy \frac{dy}{dx} = 0$$

6 **(b)**

$$\text{Given, } y = (c_1 + c_2) \cos(x + c_3) - c_4 e^{x+c_5}$$

$$\Rightarrow y = (c_1 \cos c_3 + c_2 \cos c_3) \cos x$$

$$- (c_1 \sin c_3 + c_2 \sin c_3) \sin x - c_4 e^{c_5} e^x$$

$$\Rightarrow y = A \cos x - B \sin x + C e^x$$

$$\text{Where, } A = c_1 \cos c_3 + c_2 \cos c_3$$

$$B = c_1 \sin c_3 + c_2 \sin c_3$$

$$\text{And } C = -c_4 e^{c_5}$$

Which is an equation containing three arbitrary constant. Hence, the order of the differential equation is 3.

7 **(c)**

$$\text{Given equation is } e^x + \sin\left(\frac{dy}{dx}\right) = 3$$

Since, the given differential equation cannot be written as a polynomial in all the differential coefficients, the degree of the equation is not defined.

8 **(d)**

$$\text{Given, } x = \sin t, \quad y = \cos pt$$

$$\frac{dx}{dt} = \cos t, \quad \frac{dy}{dt} = -p \sin pt$$

$$\therefore \frac{dy}{dx} = - \frac{p \sin pt}{\cos t}$$

$$\Rightarrow y_1 = \frac{-p \sqrt{1-y^2}}{\sqrt{1-x^2}}$$

$$\begin{aligned}
\Rightarrow y_1 \sqrt{1-x^2} &= -p \sqrt{1-y^2} \\
\Rightarrow y_1^2 (1-x^2) &= p^2 (1-y^2) \\
\Rightarrow 2y_1 y_2 (1-x^2) - 2x y_1^2 &= -2y y_1 p^2 \quad [\text{differentiating}] \\
\Rightarrow (1-x^2) y_2 - x y_1 + p^2 y &= 0
\end{aligned}$$

9 **(c)**

Given, $y = x e^{cx}$... (i)

$$\Rightarrow \frac{dy}{dx} = e^{cx} + x e^{cx} \cdot c = \frac{y}{x} + y \cdot c \quad \dots (ii)$$

From Eq. (ii),

$$\log y = \log x + cx$$

$$\begin{aligned}
\Rightarrow c \frac{1}{x} \log \frac{y}{x} \\
\frac{dy}{dx} &= \frac{y}{x} + \frac{y}{x} \log \frac{y}{x} \\
&= \frac{y}{x} \left(1 + \log \frac{y}{x} \right)
\end{aligned}$$

10 **(a)**

Given differential equation is

$$\begin{aligned}
y &= x \frac{dy}{dx} + \left(a^2 \left(\frac{dy}{dx} \right)^2 + b^2 \right)^{\frac{1}{3}} \\
\Rightarrow \left(y - x \frac{dy}{dx} \right)^3 &= a^2 \left(\frac{dy}{dx} \right)^2 + b^2
\end{aligned}$$

$\therefore$ Order and degree of the above differential equations are 1 and 3 respectively.

11 **(b)**

We have,

$$\begin{aligned}
y_3^{2/3} + 2 + 3y_2 + y_1 &= 0 \\
\Rightarrow y_3^{2/3} &= -(3y_2 + y_1 + 2) \\
\Rightarrow y_2^3 &= -(3y_2 + y_1 + 2)^3
\end{aligned}$$

Clearly, it is differential equation of third order and second degree

12 **(b)**

Given, $x^2 + y^2 = 1$ (i)

On differentiating w. r. t. x , we get

$$2x + 2yy' = 0 \Rightarrow x + yy' = 0$$

Again, on differentiating w. r. t. x , we get

$$1 + (y')^2 + yy'' = 0$$

13 **(a)**

We have,

$$\frac{dy}{dx} = \frac{x \log x^2 + x}{\sin y + y \cos y}$$

$$\Rightarrow \int (\sin y + y \cos y) dy = 2 \int x \log x \, dx + \int x \, dx$$

$$\Rightarrow y \sin y = x^2 \log x + C$$

14 **(b)**

The given differential equation is

$$\frac{dy}{dx} + P(x)y = Q(x).y^n$$

$$\Rightarrow \frac{1}{y^n} \cdot \frac{dy}{dx} + y^{-n+1}P(x) = Q(x)$$

Put $\frac{1}{y^{n-1}} = v$

$$\Rightarrow (-n+1)y^{-n} \frac{dy}{dx} = \frac{dv}{dx}$$

$$\therefore \frac{1}{(-n+1)} \cdot \frac{dv}{dx} + P(x).v = Q(x)$$

$$\Rightarrow \frac{dv}{dx} + (1-n)P(x)v = (1-n)Q(x)$$

Hence, required substitution is $v = \frac{1}{y^{n-1}}$

15 **(b)**

Since, length of subnormal = a

$$\Rightarrow y \frac{dy}{dx} = a \Rightarrow y dy = a dx$$

On integrating both sides, we get

$$\frac{y^2}{2} = ax + b$$

Where b is a constant of integration

$$\Rightarrow y^2 = 2ax + 2b$$

16 **(c)**

Given, $\frac{dx}{dt} = \cos^2 \pi x$

On differentiating w. r. t. x , we get

$$\frac{d^2x}{dt^2} = -2\pi \sin 2\pi x = \text{negative}$$

The particle never reaches the point, it means

$$\frac{d^2x}{dt^2} = 0 \Rightarrow -2\pi \sin 2\pi x = 0$$

$$\Rightarrow \sin 2\pi x = \sin \pi$$

$$\Rightarrow 2\pi x = \pi \Rightarrow x = \frac{1}{2}$$

The particle never reaches at $x = \frac{1}{2}$

17 **(b)**

Given, $\frac{dy}{dx} = \frac{x+y}{x-y}$

Put $y = vx \Rightarrow \frac{dy}{dx} = v + x \frac{dv}{dx}$

$$\therefore v + x \frac{dv}{dx} = \frac{1+v}{1-v}$$

$$\Rightarrow x \frac{dv}{dx} = \frac{1+v^2}{1-v}$$

$$\Rightarrow \frac{1}{x} dx = \left(\frac{1}{1+v^2} - \frac{v}{1+v^2} \right) dv$$

$$\Rightarrow \log_e x = \tan^{-1} v - \frac{1}{2} \log_e (1 + v^2) - \log_e c \quad [\text{integrating}]$$

$$\Rightarrow \log_e x = \tan^{-1} \left(\frac{y}{x} \right) - \frac{1}{2} \log_e \left[1 + \left(\frac{y}{x} \right)^2 \right] - \log_e c$$

$$\Rightarrow c(x^2 + y^2)^{1/2} = e^{\tan^{-1}(y/x)}$$

18 **(b)**

$$\text{Given, } \frac{d^2 y}{dx^2} = e^{-2x}$$

$$\Rightarrow \frac{dy}{dx} = \frac{e^{-2x}}{-2} + c \quad [\text{integrating}]$$

$$\Rightarrow y = \frac{e^{-2x}}{4} + cx + d \quad [\text{integrating}]$$

19 **(b)**

$$\text{Given, } x^2 + y^2 = 1$$

On differentiating w. r. t. x , we get

$$2x + 2yy' = 0$$

$$\Rightarrow x + yy' = 0$$

Again, differentiating, we get

$$1 + yy'' + (y')^2 = 0$$

20 **(a)**

We have,

$$\frac{dy}{dx} = \frac{y-1}{x^2+x}$$

$$\Rightarrow \frac{1}{x^2+x} dx = \frac{1}{y-1} dy$$

$$\Rightarrow \int \frac{1}{x(x+1)} dx = \int \frac{1}{y-1} dy$$

$$\Rightarrow \int \frac{1}{x(x+1)} dx = \int \frac{1}{y-1} dy$$

$$\Rightarrow \int \left(\frac{1}{x} - \frac{1}{x+1} \right) dx = \int \frac{1}{y-1} dy$$

$$\Rightarrow \log x - \log(x+1) = \log(y-1) + \log C$$

$$\Rightarrow \frac{x}{x+1} = C(y-1) \quad \dots(i)$$

This passes through (1, 0)

$$\therefore \frac{1}{2} = -C$$

Substituting the value of C in (i), we get

$$\frac{x}{x+1} = -\frac{1}{2}(y-1)$$

$$\Rightarrow (x+1)(y-1) = -2x \Rightarrow xy + x + y - 1 = 0$$

This is the required curve

ANSWER-KEY

[illegible]